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Kinetics of segregation in a two-lane highway traffic flow

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Abstract. A particle-hopping model is presented to simulate the segregation of cars on a two-lane highway which consists of a slow lane and a fast lane. The changing of cars between the slow and fast lanes is taken into account. When a fast (slow) car overtakes (is overtaken by) a slow (fast) car on the slow (fast) lane, the fast (slow) car shifts to the fast (slow) lane. By satisfying the demand for faster movement, a segregation of cars occurs. The car densities, velocities and currents on the slow and fast lanes are calculated by computer simulation. The velocity distributions on the two lanes are also shown. It is found that the traffic current is enhanced by the changing of cars between two lanes. The kinetics of segregation between the slow and fast lanes is described in terms of a Boltzmann-like kinetic equation. The kinetic equation is solved by a numerical method. The velocity distributions, car densities, velocities and currents obtained from the kinetic equation are compared with the simulation results.

1. Introduction

Recently, traffic problems have attracted considerable attention. A variety of approaches have been applied to describe the collective properties of traffic flow [1–11]. The car-following models, the cellular automaton (CA) models and the particle-hopping models are being applied successfully to simulations of traffic. When the number of cars is large, traffic flows can be modelled phenomenologically in terms of a one-dimensional compressible gas. Such a hydrodynamic approach predicts the appearance of traffic jams or traffic solitons. However, the hydrodynamic approach does not naturally describe the behaviour of traffic flows in the low-density limit where there are large heterogeneities in traffic density [7]. For this situation, such microscopic models as the CA models or the particle-hopping models will provide a more appropriate description. Furthermore, it is important to know the velocity distribution of cars moving on a multi-lane highway since each driver wants to satisfy the demand for faster movement.

Nagel and Schreckenberg [3] introduced a stochastic CA model to take into account the acceleration or deceleration of cars. They showed that the start–stop waves (traffic jams) appear in the congested traffic region as is observed in real freeway traffic. Bando *et al* [9] proposed the optimal velocity model in which a car accelerates or decelerates according to the dynamical equation of motion. Komatsu and Sasa [12] derived the modified KdV equation from the optical velocity model to describe the traffic jam.

Nagel and Herrmann [13] and Nagel [14] found that the open-boundary version of the CA traffic model exhibits a self-organized criticality, providing enough input and output of cars at the boundaries. Nagatani [15] also showed that the self-organized criticality appears in the asymmetric simple exclusion model with open boundaries where the annihilation of traffic jams is described by the ballistic annihilation process.

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In contrast to the traffic jam at high car density, car bunching occurs even at low car density. Nagatani [11] presented the stochastic particle-hopping model for the car bunching which occurs due to the difference of the inherent velocities of individual cars. The car with low velocity prevents the car with high velocity from going ahead. Cars flowing on a highway cluster more and more when moving ahead. Ben-Naim *et al* [11] analysed the kinetic clustering of cars by using a simple aggregation model. They found the scaling relationship of the kinetic clustering. It was shown that the velocity distribution of cars is important for the car-bunching phenomena.

The characteristic properties of traffic flow on a two-lane highway have been little studied for comparison with single-lane traffic. By extending the single-lane highway to the two-lane highway, car bunching is reduced by the changing of cars between the first and second lanes, and the traffic current is also enhanced by more than twice the current of a single lane. This enhancement of the traffic current is due to the segregation of cars on the slow and fast lanes. It will be important to know the characteristic properties of the two-lane highway traffic flow.

In this paper, we present the particle-hopping model to simulate the two-lane highway traffic flow. In the model, the inherent velocities of individual cars and the changing of cars between the two lanes are taken into account. Cars with high velocity tend to move on the fast lane and cars with low velocity move on the slow lane. We study this segregation of cars flowing on a two-lane highway by computer simulation. We calculate the car densities, velocities, currents and velocity distributions on the two lanes. We formulate the kinetics of segregation by a Boltzmann-like kinetic equation. We solve numerically the kinetic equation. We obtain the characteristic traffic properties and compare this result with the simulation result.

The organization of the paper is as follows. In section 2 we present the stochastic particle-hopping model of a two-lane highway. We study the traffic behaviour on a two-lane highway by computer simulation. In section 3 a simple theoretical consideration is given for a perfect segregation between slow and fast cars on a two-lane highway. In section 4 the two-lane traffic flow is formulated by a Bolzmann-like kinetic equation. The kinetic equation is solved by a numerical method. The result obtained by the kinetic equation is compared with the simulation result. Finally, section 5 contains a brief summary.

2. Model and simulation

We consider cars flowing on a two-lane highway which consists of a slow lane and a fast lane. Each car has its own inherent velocity. We assume that the car velocities are limited between the maximal and minimal velocities. The maximal velocity is controlled by the cars' performance. The minimal velocity is governed by the limit of lower velocity which is determined by public demand. If a car is not blocked ahead by another car, the car moves with its inherent velocity. However, if a car is blocked ahead by a slow car, it moves with the same velocity as the slow car. Each driver wants to satisfy the demand for faster movement. When a fast car overtakes a slow car on the slow lane and the velocity of the overtaking car is larger than the shifting velocity v_c, the fast car shifts to the fast lane. On the other hand, if a slow car is overtaken by a fast car on the fast lane, the slow car shifts to the slow lane. Then a segregation of cars occurs: fast cars gather on the fast lane and slow cars are on the slow lane. Without changing lane, car bunching or clustering occurs. Then the traffic current is controlled by the slowest car. However, by allowing cars to change lanes, the traffic current is enhanced upon that without changing lanes.

We try to simulate the traffic flow by as simple a model as possible. We extend the one-dimensional fully asymmetric simple exclusion model with parallel update to take into account both the inherent velocity of cars and the changing of cars between the two lanes. The particle-hopping model is defined on two one-dimensional lattices of $2 \times L$ sites with periodic boundary conditions. Each site is occupied by a single car or is empty. On odd time steps, car i on the slow lane moves ahead by one step with the inherent hopping probability p_i unless car i is blocked ahead by another car. If car i is blocked ahead by another car, its hopping probability p_i is larger than the shifting velocity v_c and its nearest-neighbour site on the fast lane is unoccupied, car i shifts to its nearest-neighbour site on the fast lane. On odd time steps one update of the system for an arbitrary configuration is performed in parallel for all cars on the slow lane. On even time steps, car j on the fast lane moves ahead by one step with the inherent hopping probability p_j unless car j is blocked ahead by another car and car j is overtaken by another car. When car j on the fast lane is blocked ahead by another car, car j does not move and remains on the site. If car j on the fast lane is overtaken by another car and its nearest-neighbour site on the slow lane is unoccupied, car j shifts to its nearest-neighbour on the slow lane. On even time steps one update of the system for an arbitrary configuration is performed in parallel for all cars on the fast lane. Figure 1(a) shows schematically the rules of the model. The top traffic configuration changes to the bottom pattern after two time steps. The car indicated by the double circle represents the car overtaking a slow car on the slow lane (lane 1). After the next time step, the car shifts to the fast lane (lane 2) vertically. The car indicated by the square-dot point represents the car being overtaken by a fast car on the fast lane (lane 2). After the next time step, the car shifts to the slow lane (lane 1) vertically. In the limit of no changing lane, our particle-hopping model is reduced to the model on the single-lane highway.

The car velocity is taken into account as the hopping probability in the particle-hopping model. The car velocity v is normalized by the maximal velocity $v_{\max}$. The normalized velocity $v/v_{\max}$ is limited between $v_{\min}/v_{\max}(< 1)$ and 1 where $v_{\min}$ is the minimal velocity. In the dilute limit of car density, car i moves ahead with the mean velocity p_i at coarse-grained time scales since car i is little blocked by another car. The hopping probability p_i corresponds to the inherent velocity v_i of individual cars where v_i is normalized and dimensionless. The limiting value of the hopping probability for changing lane corresponds to the shifting velocity v_c where v_c is also normalized by the maximal velocity. Car bunching occurs due to the difference of inherent velocities of individual cars. A car moving with low velocity prevents a car moving with high velocity from going ahead. Cars flowing on a highway cluster more and more. By introducing the change of cars between the slow and fast lanes, car bunching is reduced by the segregation effect which is induced by changing lane. The traffic current is enhanced by reducing car bunching.

In this model, the traffic problem on a highway is reduced to its simplest form while the essential features are maintained. The feature includes the flow in one direction of cars which cannot overlap. Furthermore, this model possesses both properties that the car moving with low velocity prevents the car moving with high velocity from going ahead and that the car on the slow (fast) lane shifts to the fast (slow) lane when the fast (slow) car overtakes (is overtaken by) the slow (fast) car on the slow (fast) lane.

We perform the computer simulation for the above model. Initially, cars are randomly distributed on the sites of two one-dimensional lattices with car density ρ. Furthermore, the hopping probability p_i is assigned to each car. The hopping probability p_i assigned to each car does not change with time. We assume that the hopping probability p_i is uniformly distributed between a and b. We set $a = 0.5$ and $b = 1.0$ where a is the normalized minimal velocity and b is the normalized maximal velocity. The simulations are performed for the system size $L = 10^4$ and the car density $\rho = 0.1$. In the limiting case of no changing lanes,

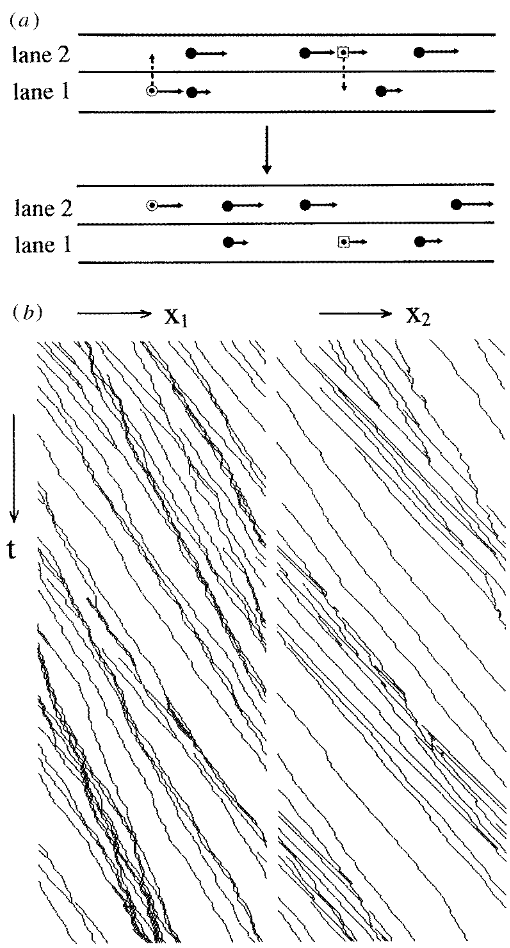

Figure 1. (*a*) The lane-changing rules are shown schematically. The top traffic configuration changes to the bottom pattern after two time steps. The car indicated by double circles represents the car overtaking a slow car on the slow lane (lane 1). After the next time step, the car shifts to the fast lane (lane 2). The car indicated by the square-dot point represents the car being overtaken by a fast car on the fast lane (lane 2). After the next time step, the car shifts to the slow lane (lane 1). (*b*) A typical spacetime pattern of cars for car density $\rho = 0.1$ and shifting velocity $v_c = 0.7$ up to 500 time steps where the system size is $L = 190$. The pattern on the right-hand side represents the spacetime configuration on the fast lane. The pattern on the left-hand side represents the spacetime configuration on the slow lane. The horizontal direction indicates that in which cars move ahead. The vertical direction is that of time. A trajectory of a single car is represented by a curve. Fast cars gather on the fast lane and slow cars are on the slow lane.

the typical headway $\langle s \rangle$ scales as $\langle s \rangle \approx t^{0.5}$ for values less than $\rho = 0.1$. With increasing car density, the typical headway is saturated to a constant value by the excluded effect. In order to study the traffic flow on a two-lane highway at low density, we set the car density

$\rho = 0.1$ Each run is calculated until 10^5 time steps. For illustration, figure 1(*b*) shows a typical pattern of cars for the car density $\rho = 0.1$ and the limiting velocity $v_c = 0.7$ up to 500 time steps where the system size is $L = 190$. The pattern on the right-hand side represents that on the fast lane. The pattern on the left-hand side represents that on the slow lane. The horizontal direction indicates that in which cars move ahead. The vertical direction is that of time. A car is indicated by a dot. The trajectory of a car is represented by a curve. Fast cars gather on the fast lane and slow cars are on the slow lane. Segregation occurs between the two lanes. For comparison with single-lane highway traffic, car bunching is reduced by the segregation. In the limit of the shifting probability $p_{\mathrm{exc}} = 0$, the typical headway shows power law behaviour. The limit corresponds to the case without changing lane. Then the traffic current is controlled by the slowest car. Car bunching occurs. However, by allowing cars to change lane, the traffic current is not limited by the slowest car. Then the traffic current is enhanced. In the present model, the shifting probability p_{exc} equals one. Then the scaling behaviour of the typical headway breaks down. The enhancement of the traffic current is shown quantitatively in figure 4.

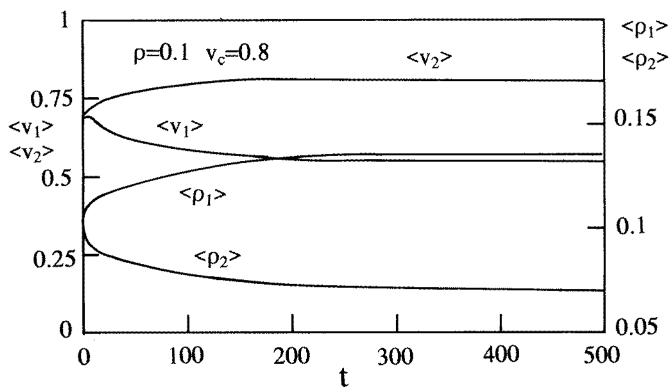

Figure 2. The plot of mean velocities $\langle v_1\rangle$, $\langle v_2\rangle$ and mean densities $\langle \rho_1\rangle$, $\langle \rho_2\rangle$ on slow and fast lanes against time t for shifting velocity $v_c = 0.8$ and density $\rho = 0.1$. The segregation between slow and fast lanes occurs with increasing time.

Figure 2 shows the plot of mean velocities $\langle v_1\rangle$, $\langle v_2\rangle$ and mean densities $\langle \rho_1\rangle$, $\langle \rho_2\rangle$ on the slow and fast lanes against time t for the shifting velocity $v_c = 0.8$. With increasing time, segregation occurs: the mean velocity $\langle v_1\rangle$ of cars on the slow lane decreases and the mean velocity $\langle v_2\rangle$ of cars on the fast lane decreases. After 500 time steps, the mean velocities and the densities become constant values of a steady state. Figure 3 shows the plot of steady-state values of mean velocities $\langle v_1\rangle$, $\langle v_2\rangle$ and mean densities $\langle \rho_1\rangle$, $\langle \rho_2\rangle$ against the shifting velocity v_c. With increasing shifting velocity v_c, the velocity $\langle v_2\rangle$ of the fast lane increases, and in constrast, the density $\langle \rho_2\rangle$ of the fast lane decreases. In figure 4, we show the plot of traffic currents $\langle J\rangle$, $\langle J_1\rangle$ and $\langle J_2\rangle$ against the shifting velocity v_c where $\langle J\rangle$, $\langle J_1\rangle$ and $\langle J_2\rangle$ are, respectively, the total traffic currents ($\langle J\rangle = \langle J_1\rangle + \langle J_2\rangle$), the traffic current of the slow lane and the traffic current of the fast lane. With increasing shifting velocity v_c, current $\langle J_2\rangle$ of the fast lane decreases. In contrast, current $\langle J_1\rangle$ of the slow lane increases with shifting velocity v_c. Total current $\langle J\rangle$ remains a constant value until shifting velocity $v_c = 0.8$. Without changing lanes, the traffic current is controlled by the slowest car. Without changing lanes, the total current equals twice $\langle J\rangle = 0.1$ of the single-lane current $\langle J_1\rangle = \langle \rho_1\rangle v_{\mathrm{min}} = 0.1 \times 0.5$. With changing lanes, the total current $\langle J\rangle$ increases by about 30%. This enhancement of the current is important in traffic technology. The

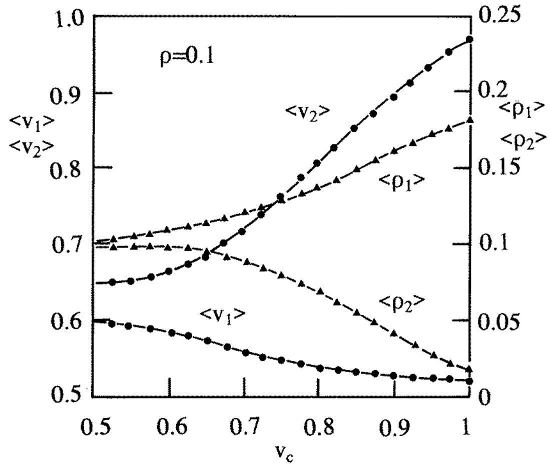

Figure 3. The plot of steady-state values of mean velocities $\langle v_1 \rangle$, $\langle v_2 \rangle$ and densities $\langle \rho_1 \rangle$, $\langle \rho_2 \rangle$ against shifting velocity v_c for density $\rho = 0.1$.

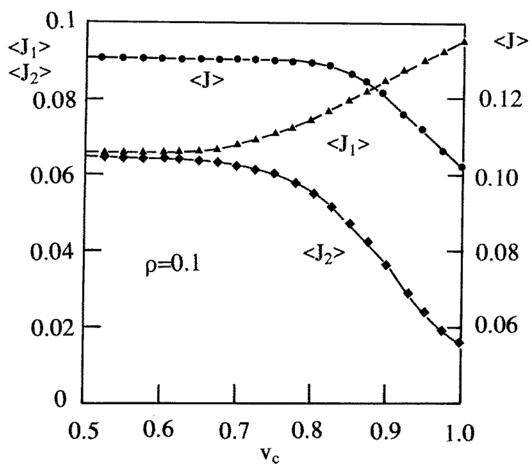

Figure 4. The plot of total traffic current $\langle J \rangle$, slow-lane current $\langle J_1 \rangle$ and fast-lane current $\langle J_2 \rangle$ against shifting velocity v_c for density $\rho = 0.1$.

enhancement is due to the car segregation between the fast and slow lanes. The traffic current increases even in the limit of $v_c = 0.5$ where the cars on the slow lane shift to the fast lane if any cars overtake the slow car.

Figures 5 and 6 show the inherent velocity distributions on the slow and fast lanes for shifting velocities $v_c = 0.5$ and 0.6 where the car density $P(v)$ with the inherent velocity v is plotted with the interval $\Delta v = 0.01$. The fast cars gather on the fast lane and the slow cars gather on the slow lane. By this segregation, each car satisfies the demand for faster movement.

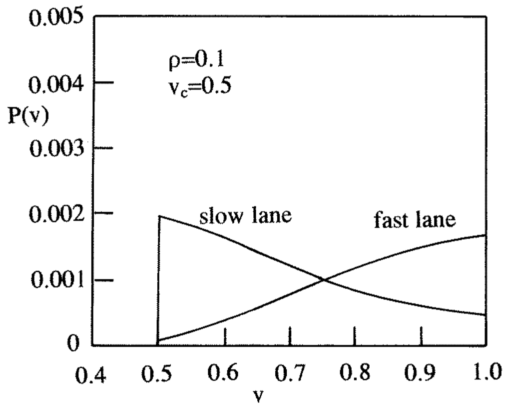

Figure 5. The inherent velocity distributions $P(v)$ on the slow and fast lanes for shifting velocity $v_c = 0.5$ and density $\rho = 0.1$.

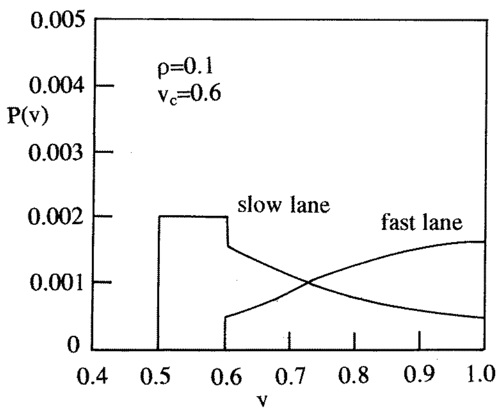

Figure 6. The inherent velocity distributions $P(v)$ on the slow and fast lanes for shifting velocity $v_c = 0.6$ and density $\rho = 0.1$.

3. Perfect segregation

We consider the limiting case of a perfect segregation in which the cars faster than shifting velocity v_c shift to the fast lane and the cars slower than shifting velocity v_c remain on the slow lane. After the fast cars shift to the fast lane, the cars on the fast lane do not shift to the slow lane. In this case, the traffic properties are calculated easily. The velocity of the slow lane is controlled by the slowest car on the slow lane: $v_1 = 0.5$. The velocity of the fast lane is also limited by the slowest car on the fast lane: $v_2 = v_c$. The densities on the slow and fast lanes are given by $\rho_1 = 4(v_c - \frac{1}{2})\rho$ and $\rho_2 = 4(1 - v_c)\rho$. The traffic currents are obtained: $J_1 = \rho_1 v_1 = 2(v_c - \frac{1}{2})\rho$, $J_2 = \rho_2 v_2 = 4v_c(1 - v_c)\rho$ and $J = J_1 + J_2 = (-1 + 6v_c - 4^2 v_c)\rho$ where J_1, J_2, and J are the slow-lane current, the fast-lane current and the total current. Figure 7 shows the plot of the velocities v_1, v_2 and the densities ρ_1, ρ_2 against shifting velocity v_c for $\rho = 0.1$. Figure 8 shows the plots of the currents J_1, J_2 and J against v_c. By comparing the results with the simulation results in figures 3 and 4, the densities ρ_1 and ρ_2 deviate largely at low and intermediate densities from those in figure 3. Also, the traffic currents J_1 and J_2 deviate largely at low and intermediate densities from those in figure 4. The total current J increases with shifting velocity v_c at low density, reaches the maximal value $J = 0.12$ at $v_c = 0.75$, and then decreases at high density. The maximal current increases by about 20 percent more than twice the single-lane current. The traffic behaviour of the perfect segregation is definitely different from the simulation results. Therefore, the two-lane traffic properties cannot be

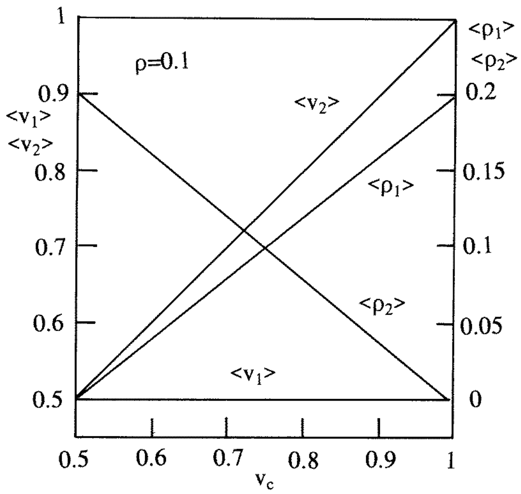

Figure 7. The plot of velocities v_1, v_2 and densities ρ_1, ρ_2 against shifting velocity v_c for density $\rho = 0.1$ in the case of perfect segregation.

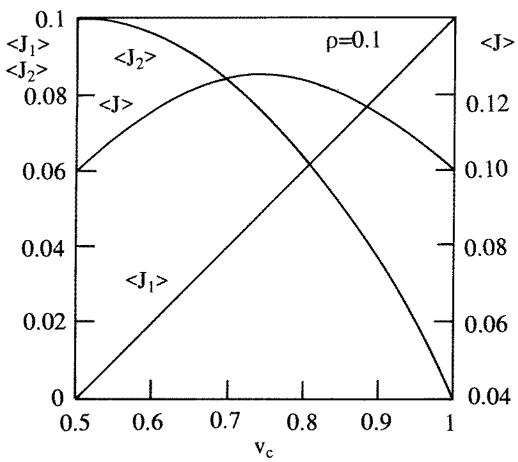

Figure 8. The plot of slow-lane current J_1, fast-lane current J_2 and total current $J(= J_1 + J_2)$ against shifting velocity v_c for density $\rho = 0.1$ in the case of perfect segregation.

described by the perfect segregation. It is necessary to describe the traffic segregation by the kinetic equation.

4. Kinetic equations

We describe the kinetics of car segregation in terms of Boltzman-type kinetic equations. We denote the density $P_1(v, t)$ $(P_2(v, t))$ of cars on the slow (fast) lane with velocity v at time t. In the slow lane, the loss of v cars due to collisions with slower v' cars occurs at a rate proportional to the relative velocity, $(v - v')$. We assume that the pair correlation function factorizes into a product of single-car velocity distributions, $P_1(v, v', t) = P_1(v, t)P_1(v', t)$. If the fast car on the slow lane overtakes the slow car and the nearest-neighbour site on the fast lane is unoccupied, the fast car on the slow lane shifts to the fast lane. The probability that a site on the fast lane is unoccupied is given by

$$\left\{1 - \int_{v_{\min}}^{v_{\max}} \mathrm{d}v''\, P_2(v'', t)\right\}.$$

Thus, the loss of v cars is given by the first term of equation (1) below. Similarly, in the fast lane, the loss of v cars due to collisions with faster v' cars occurs at a rate proportional to the relative velocity, $(v' - v)$. When the slow car on the fast lane is overtaken by the fast car and the nearest-neighbour site on the slow lane is unoccupied, the slow car on the fast lane shifts to the slow lane. Thus, the loss of v cars is given by the second term equation (2) below. Since the cars shift to the other lane by collisions, the loss of v cars on the slow (fast) lane becomes the product of v cars on the fast (slow) lane. For cars faster than the limiting velocity v_c, the velocity distributions $P_1(v, t)$ and $P_2(v, t)$ on the slow and fast lanes, respectively, described by

$$\begin{aligned}\frac{\partial P_1(v,t)}{\partial t} &= -\left\{1 - \int_{v_{\min}}^{v_{\max}} \mathrm{d}v''\, P_2(v'',t)\right\} P_1(v,t) \int_{v_{\min}}^{v} \mathrm{d}v'\,(v - v') P_1(v',t) \\ &\quad + \left\{1 - \int_{v_{\min}}^{v_{\max}} \mathrm{d}v''\, P_1(v'',t)\right\} P_2(v,t) \int_{v}^{v_{\max}} \mathrm{d}v'\,(v' - v) P_2(v',t) \qquad (1)\end{aligned}$$

$$\begin{aligned}\frac{\partial P_2(v,t)}{\partial t} &= +\left\{1 - \int_{v_{\min}}^{v_{\max}} \mathrm{d}v''\, P_2(v'',t)\right\} P_1(v,t) \int_{v_{\min}}^{v} \mathrm{d}v'\,(v - v') P_1(v',t) \\ &\quad - \left\{1 - \int_{v_{\min}}^{v_{\max}} \mathrm{d}v''\, P_1(v'',t)\right\} P_2(v,t) \int_{v}^{v_{\max}} \mathrm{d}v'\,(v' - v) P_2(v',t) \qquad (2)\end{aligned}$$

where $v_{\min}$ and $v_{\max}$ are the minimal and maximal velocities. Equations (1) and (2) hold for the cars faster than shifting velocity v_c. On the slow lane, the cars slower than shifting velocity v_c do not shift to the other lane. The slow car prevents the fast cars from going ahead. The car velocity is controlled by the slowest car. The slow cars on the slow lane induce car bunching or clustering. Ben-Naim *et al* [1] derived the governing equation for the car bunching beyond the mean-field approximation. The velocity distribution on the slow lane is given by

$$\frac{\partial P_1(v,t)}{\partial t} = -P_1(v,t) \int_{v_{\min}}^{v} \mathrm{d}v'\,(v - v') P_1(v',0) \qquad (3)$$

for $v_{\min} < v < v_c$. Here, the pair correlation function factorizes into a product of single-particle velocity distributions, $P_1(v, v', t) = P_1(v, t)P_1(v', 0)$ but with different time arguments for the two factors. In contrast, in the conventional Boltzmann equation, the decomposition would involve the same argument for each velocity distribution. Thus the exact equation (3) quantitatively indicates the degree of approximation of the mean-field Boltzmann equation. The car density $P_1(v_{\min}, t)$ of the slowest car is given by

$$\frac{\partial P_1(v_{\min},t)}{\partial t} = P_1(v_{\min},t) \int_{v_{\min}}^{v_{\max}} \mathrm{d}v'\,(v' - v_{\min}) P_1(v',0). \qquad (4)$$

Since the car on the fast lane can shift to the slow lane, the velocity distribution on the fast lane can be described by equation (2) for $v_{\min} < v < v_{\max}$.

We solve equations (1)–(4) simultaneously by the numerical method in which the integration is replaced by the summation with the interval $\Delta v = 0.01$ and the time derivative is replaced by the time difference, $P(v, t + 1) - P(v, t)$. Figure 9 shows the velocity distributions $P_1(v, \infty)$ and $P_2(v, \infty)$ in the steady state after sufficiently large times for shifting velocity $v_c = 0.5$. The slow cars tend to gather on the slow lane, and in contrast, the fast cars tend to gather on the fast lane. The velocity distributions are compared with those in figure 5 obtained by the model simulation. The velocity distributions obtained by the Boltzman-type equations agree well with the results in figure 5. Also, figure 10 shows the velocity distributions of the steady state for limiting velocity $v_c = 0.7$.

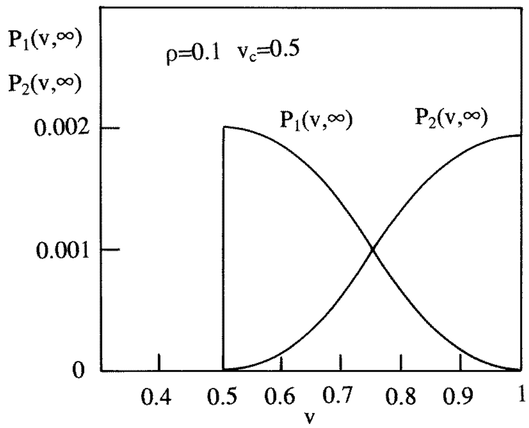

Figure 9. The velocity distributions $P_1(v, \infty)$ and $P_2(v, \infty)$ on the slow and fast lanes in the steady state for shifting velocity $v_c = 0.5$ and density $\rho = 0.1$.

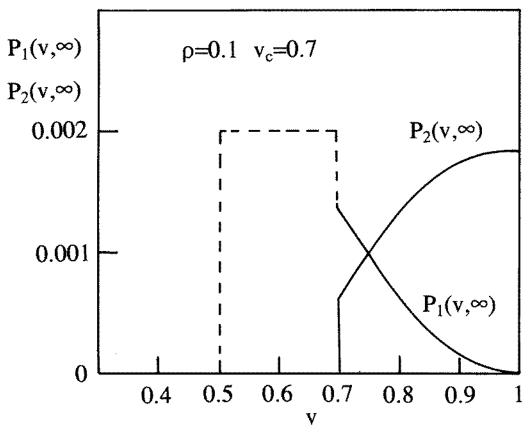

Figure 10. The velocity distributions $P_1(v, \infty)$ and $P_2(v, \infty)$ on the slow and fast lanes in the steady state for shifting velocity $v_c = 0.7$ and density $\rho = 0.1$.

We calculate the mean velocities $\langle v_1\rangle$, $\langle v_2\rangle$, the mean densities $\langle \rho_1\rangle$, $\langle \rho_2\rangle$, and the mean traffic current $\langle J_1\rangle$, $\langle J_2\rangle$, $\langle J\rangle (= \langle J_1\rangle + \langle J_2\rangle)$. The mean velocity $\langle v_1\rangle$, the mean density $\langle \rho_1\rangle$ and the mean current $\langle J_1\rangle$ are defined by

$$\langle v_1\rangle \equiv \int_{v_{\min}}^{v_{\max}} \mathrm{d}v\, v P_1(v, \infty) \Big/ \int_{v_{\min}}^{v_{\max}} \mathrm{d}v\, P_1(v, \infty)$$

$$\langle \rho_1\rangle \equiv \int_{v_{\min}}^{v_{\max}} \mathrm{d}v\, P_1(v, \infty)$$

and

$$\langle J_1\rangle \equiv \int_{v_{\min}}^{v_{\max}} \mathrm{d}v\, v P_1(v, \infty).$$

Figure 11 shows the plot of mean velocities $\langle v_1\rangle$, $\langle v_2\rangle$ and mean densities $\langle \rho_1\rangle$, $\langle \rho_2\rangle$ against velocity v_c. The density $\langle \rho_1\rangle$ on the slow lane increases with shifting velocity v_c and on the other hand, the density $\langle \rho_2\rangle$ decreases with increasing v_c. Also, the mean velocity $\langle v_1\rangle$ on the slow lane decreases with increasing v_c since car bunching occurs. The results are compared with those in figure 3 obtained by the model simulation. The values of $\langle v_1\rangle$, $\langle \rho_1\rangle$ and $\langle \rho_2\rangle$ agree well with the simulation results in figure 3. However, the velocity $\langle v_2\rangle$ on the fast lane deviates from the simulation result. This is due to neglecting the car bunching on the fast lane in kinetic equation (2). Figure 12 shows the plot of the traffic currents $\langle J_1\rangle$, $\langle J_2\rangle$ and $\langle J\rangle$

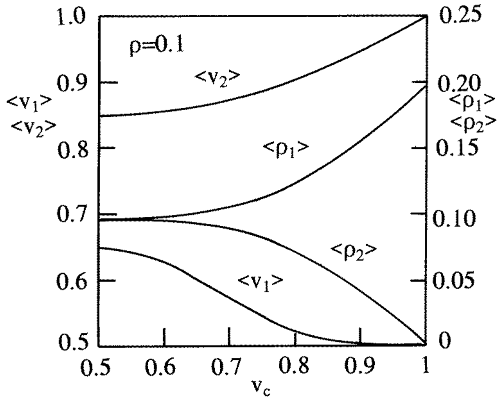

Figure 11. The plot of mean slow-lane velocity $\langle v_1 \rangle$ mean fast-lane velocity $\langle v_2 \rangle$, mean slow-lane density $\langle \rho_1 \rangle$ and mean fast-lane density $\langle \rho_2 \rangle$ against shifting velocity v_c for density $\rho = 0.1$.

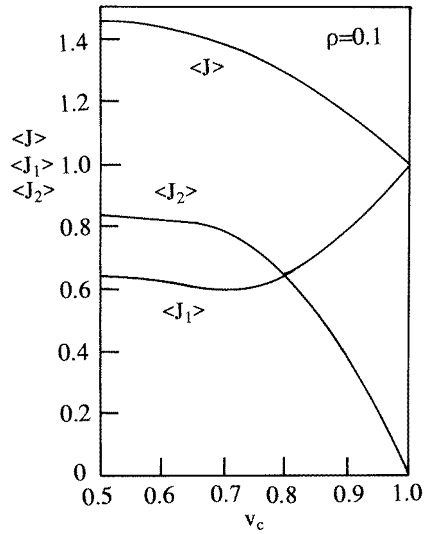

Figure 12. The plot of mean slow-lane current $\langle J_1 \rangle$, mean fast-lane current $\langle J_2 \rangle$ and mean total current $\langle J \rangle (= \langle J_1 \rangle + \langle J_2 \rangle)$ against shifting velocity v_c for density $\rho = 0.1$.

against shifting velocity v_c. The results are compared with those in figure 4 obtained by the model simulation. The traffic current $\langle J_2 \rangle$ on the fast lane is larger than the result in figure 4. This is due to the overestimate of the velocity $\langle v_2 \rangle$ on the fast lane. Thus, the total current $\langle J \rangle$ is larger than the result in figure 4. This may be due to the approximation of the pair correlation function factorizing into a product of single-car velocity distributions. The values of the traffic properties calculated by the kinetic equations are consistent with the simulation results except for the velocity on the fast lane. Furthermore, in order to obtain more accurate values, it will be necessary to take into account the car bunching on the fast lane.

5. Summary

We have studied the traffic flow on a two-lane highway by the simulation model. We have computed the velocity distributions of two lanes, and the mean traffic properties (density, velocity and current). We have shown that a car segregation between the slow and fast lanes occurs. Particular emphasis has been paid to the velocity distributions. We have found that lane changing induces an enhancement of the traffic current. Furthermore, we have investigated the kinetics of the traffic flow on a two-lane highway by the use of Boltzmann-type kinetic equations. We have computed the velocity distributions of the two lanes by solving numerically the kinetic equations. We have compared the velocity distributions obtained by the kinetic equation with those by the simulation model. We have shown that the traffic properties agree well with the simulation results except for the mean velocity of the fast lane.

The two-lane traffic model introduced in this paper leads to behaviour definitely different from a single-lane traffic flow. The behaviour is due to taking into account lane changing. The enhancement of the traffic current found in our two-lane model will be important in traffic technology. In order to describe traffic flow more realistically, it will be necessary to extend the two-lane traffic model to multi-lane traffic.

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